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ABSTRACT

Models for sire evaluation by best linear unbiased prediction procedures which consider all lactation records, an environmental correlation among paternal half-sisters in the same herd, herd-year-season effects, genetic groups, and sires within groups are described. The computing algorithms are outlined briefly. Attention is drawn to a mimeographed publication of 46 pages (12) which describes in detail the computing algorithms for a model including an interaction between sire and herd (the environmental correlation) and a model which does not include the environmental correlation.

INTRODUCTION

The best linear unbiased prediction (BLUP) procedure used in New York (1, 3, 7) for evaluation of about 3500 sires uses only first lactation records on artificially-sired (AI) daughters. The model is

$$y_{ijkl} = g_i + s_{ij} + h_k + e_{ijkl}$$

where y_{ijkl} is the 305-day age-adjusted record for the lth cow from the jth sire randomly drawn from the ith fixed genetic sire group freshening in the kth fixed herd-year-season. The variance ratio σ_e^2/σ_s^2 is known. Known relationships among the sires also are used in determining the covariance matrix of the s_{ij} (6).

Application of BLUP to national sire evaluation for data from USDA is considerably more complex in several respects. The first complexity is a responsibility to evaluate natural service (NS) as well as AI sires, which could result in evaluation of as many as 30,000 sires. Second, there is the tradition of using all lactation records. Records additional to the first would be expected to contribute to increased accuracy, but computing costs may be increased significantly. Incorrect age adjustment or repeatability, however, could bias prediction of genetic merit for first lactation production. Third, evidence (9, 10) suggests environmental correlation among paternal half-sisters in the same herd or herd-year-season should be considered.

Sire evaluation by BLUP methods requires the solution of a large set of simultaneous linear equations. Except perhaps for a single herd, it is impossible to keep all equations in core storage of a computer and solve by direct methods such as inverting the coefficient matrix and postmultiplying by the right-hand sides. Fortunately, most problems of sire evaluation can be formulated with a model that leads to feasible strategies for solution. Computing algorithms for several models are available for sire evaluation by BLUP (1, 2, 3, 7, 8) which make use of mixed model equations. The USDA herdmate procedure and its replacement, the modified contemporary comparison (MCC), both account for environmental correlation among paternal half-sisters in the same herd. The BLUP procedures can account for environmental correlation as a correlation among error terms reflected by a nondiagonal structure of the error variance-covariance matrix or by incorporating a separate random effect in the model. The relationship between the two equivalent models is described by Henderson (4).

Whether sire-by-herd interaction or sire-by-herd-year-season interaction better describes the environmental correlation among paternal half-sisters is not clear. Sire-by-herd interaction currently is used by the USDA and is much easier to handle than sire-by-herd-season interaction in computing sire evaluations by BLUP.

Computing algorithms for cases when

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natural service sires are evaluated, when all lactations are used, and when an environmental correlation or equivalently a sire-by-herd or herd-year-season interaction is included are complicated. The purpose of this note is to describe briefly algorithms which have been derived for those situations. The complete description of the computing procedures runs 46 pages (12) and is available from the Department of Animal Science, Cornell University, Ithaca, NY 14853.

Algorithm for Model without Environmental Correlation (Sire-by-Herd Interaction)

The model without interaction is:

$$y_{fijklm} = g_{fi} + s_{fij} + h_{kl} + c_{fijkm} + e_{fijklm}$$

where the subscript:

- f indicates the type of sire (f = 1 for AI and f = 2 for NS or absorbed sires),
- i indicates the sire group,
- j indicates the sire within the ith group and the fth type,
- k indicates the herd,
- l indicates the year-season of freshening within the kth herd,
- m indicates the cow within the fijth sire and the kth herd,

and

- y is an age-season adjusted record,
- g is a fixed sire group effect,
- s is a random sire-within-group effect with variance σ_s^2,
- h is a fixed herd-year-season (HYS) effect,
- c is random cow-within-sire effect with variance σ_c^2, and
- e is a random residual effect with variance σ_e^2.

All random effects are mutually uncorrelated. For computational simplicity only one record was accepted in any year-season for a given cow, and cows were nested within a herd.

Details of the computing algorithm are in the mimeograph series (12). The basic strategy, however, is to eliminate most of the equations by sequential absorption of cow equations, NS sire equations, and herd-year-season equations to leave AI sire equations and group equations for AI and NS sires. For the purpose of absorption, an approximation is made in that the NS sire equations as well as the cow equations are considered as nested within a herd so that their absorption results in a diagonal block structure in the herd-year-season equations. Absorption of the herd-year-season equations is accomplished one herd at a time and involves inverting each block with order of the number of year-seasons in the herd. The group and sire equations can be solved by modified Gauss-Siedel iteration. Back solutions for herd-year-seasons and then for NS sires can be obtained.

Algorithms for Model with Environmental Correlations (Sire-by-Herd Interaction)

The model with interaction is:

$$y_{fijklm} = g_{fi} + s_{fij} + h_{kl} + (sh)_{fijk} + c_{fijkm} + e_{fijklm}$$

where the subscripts and effects are as described for the no-interaction model with the addition of an effect, sh, for the sire-by-herd interaction. The variance ratios, σ_e^2/σ_s^2, σ_s^2/σ_{sh}^2, and σ_e^2/σ_c^2, will be different from when sire-by-herd interaction is not included.

Cow equations are absorbed as in the model without interaction. Sire-by-herd equations effectively are nested within herds and can be absorbed just as were NS sire equations. The back solutions for HYS and NS sire effects are obtained as described for the model without sire-by-herd effects, but if solutions for cow effects are desired, the sire-by-herd equations must be solved first.

Alternatives for the Environmental Correlation Model

The choice of an appropriate model for sire evaluation depends heavily on the computational difficulties involved. Several alternatives may be possible for dealing with the environmental correlation among paternal half-sisters.

One alternative is to include a sire-by-herd interaction as described by the preceding model and which appears the simplest in terms of computations. A second alternative is to include an effect for sire-by-herd-year-season interaction. In this case cows are cross-classified with the interaction effect. Absorption of the cow equations results in a block diagonal structure in the interaction equations as well as in the HYS equations so that two matrix inversion operations per herd are required rather than the one required for the sire-by-herd model.

There is no attempt in the mimeo series (12) to consider the merit of various alternatives in terms of fit to the data. It does appear important to study further the environmental correlation in BLUP procedures. The main consequence of using the wrong sire-by-herd variance (or erroneously including or omitting it from the model) is that the error variances of prediction will be larger than necessary. However some biases may be introduced by selection when wrong variances are used (5, 11).

Use of Relationships among AI Sires

Use of relationships among the animals was not considered. Relationships among the AI sires, however, can be handled by the procedure outlined in (6). The inverse of the matrix of numerator relationships among the AI sires multiplied by the scalar σ_e^2/σ_s^2 is added to the coefficients of the sire equations after absorption rather than an identity matrix multiplied by the scalar σ_e^2/σ_s^2 for either the model with interactions or the model with no interactions. Such a procedure would ignore relationships among NS sires and relationships among cows except when the relationship is through common AI sires.

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